



MODELLING THE DAILY AVERAGE TEMPERATURE AND DESIGNING A GROWING DEGREE DAY (GDD) EUROPEAN PUT OPTION FOR RICE IN LAGUNA

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ABSTRACT –Weather conditions are primary influencer on rice production, causing unpredictable yield. Any unexpected variation and irregularity in the weather, such as fluctuation of temperature, may contribute to reduction of rice production. Weather derivatives are financial instruments that can be used to financially protect production risks related to weather variability. Thus, the objective of this study is to price a European put option, a type of weather derivative, in the province of Laguna, Philippines, with growing degree day (GDD), as the underlying index. Specifically, this study aims to find a differential equation model of the daily average temperature and to find appropriate GDDs as strike values for the contract. Daily minimum and maximum temperatures recorded from year 1960 to 2018 are collected. It is found that the average temperature in Laguna within the specified period is 26.88°C, and varies by 1.42°C. It is also found that temperature is expected to increase by 1.72°C in 100 years assuming that current situations persist. Using this information, the GDD option is priced. It is found that by paying choosing 1800 GDD and paying 0.13 PhP, the farmer can gain as much as 321.08 PhP if he exercises the option. The results of this study can help insurance providers and the government to design products that can help the farmers. This product is relatively cheaper than other financial contracts hedging against adverse weather conditions.

Keywords: European option, growing degree day, temperature model, time series, weather derivative

INTRODUCTION

The Philippines experiences high temperature and high atmospheric humidity as a tropical country, and intense weather conditions due to climate change. There are days when the heat index reaches more than 40°C (Philippine Atmospheric, Geophysical and Astronomical Services Administration (PAGASA), 2017). Furthermore, maximum and minimum temperatures have risen by 0.36°C and 1.0°C, respectively, over the last 60 years (PAGASA, 2011).

Agriculture is one of the major industries affected by such intense weather. Rice production, for example, is highly dependent on weather. Global mean rice yields are expected to be reduced by $3.2 \pm 3.7\%$ per 1°C increase in global temperature (Stuecker *et al.* 2018). Extreme temperature makes it difficult

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for rice to develop. According to Yoshida (1981), a continued temperature of 12°C for 6 days inhibits rice crop from reaching the flowering stage, while a temperature lower than 22°C and higher than 35°C during the reproductive stage causes crop sterility. Basso and Naglekirk (2014) also supports that temperature increase affects the crop adversely by minimizing the growth time, causing reduced yield. An increase in the temperature reduces the crop yield by as much as 10% (Stuecker *et al.* 2018).

In order to manage the risks associated to these extreme weather events, financial contracts can be employed to protect farmers against undesirable financial consequences, such as the weather-indexed derivative. A derivative is a financial contract that derives its value from values of another underlying commodity. Weather derivatives are financial instruments whose value depend on a weather index and can be used to reduce the risk of loss resulting from that weather conditions (Yang, 2010). For this kind of derivative, the common indices are temperature, rainfall, snowfall, and wind.

One type of a derivative is an option: a financial contract that gives the holder the right but not the obligation to buy or sell the underlying commodity. There are two forms of options, namely, call option and put option. Call option (put option) gives the holder the right, but not the obligation, to buy (sell) the underlying asset at a pre-specified price and period. A European put option, in particular, is a type of option that can be exercised at the maturity, that is, at the end of the number of days specified in the contract (Hull, 2012).

There have been many studies on producing weather-index derivatives for agriculture. The most popular studies are the weather-indexed insurance (WIBI): India has piloted and implemented WIBI difference indices such as rainfall, consecutive dry days, untimely rainfall, frost and high temperature (Clarke, 2014); Malawi has also drought WIBI for groundnut (Global Facility for Disaster Reduction and Recovery, 2011); Morocco has drought WIBI for cereals (Skees, 2001); the Philippines through the Philippine Crop Insurance Corporation has also pilot-tested WIBI for Ilo-ilo and Cagayan (Philippine Crop Insurance Corporation, 2015).

European put option can be studied to create a new financial contract to protect the farmers against the effects of intense temperatures. With growing degree day (GDD) as the index in consideration, the objective of this study is to design a European put option for rice crops. Specifically, this study aims to model using differential equations the daily average temperature in the province of Laguna; to compute for the GDD; to find the appropriate GDD strike value to use; and, to give the premium price for the contract.

This study presents an initial study on a new form of index, which is GDD. The most common indices are rainfall and drought. There have been other studies that use heating-degree-days (HDD) and cooling-degree-days, (Alaton, 2002) but the underlying indices are not agricultural products.

The next sections discuss the methods followed to conduct this study and the results of the study. This study assumes a 100-day contract, which is the number of days to maturity from transplanting of an early maturing rice variety. The base temperature is assumed to be 12°C as below this temperature, crop growth is inhibited according to Yoshida (1981).

METHODOLOGY

The following were the major steps followed in the conduct of this study:

1. Data collection

Daily maximum and minimum temperatures recorded from 1960 to 2018 were collected. Missing data on November 2-3, 1995 and July 15-31, 2014 were supplied using cubic interpolation. After which, the daily mean or average temperature, given by, $T_i = \frac{T_i^{max} + T_i^{min}}{2}$ (1)

per day were calculated.

2. Modeling the temperature

The average temperature T_t in day t was modelled as

$$T_t = T_t^m + \alpha(T_{t-1} - T_{t-1}^m) + \sigma_t \varepsilon_t \tag{2}$$

where α is the speed of mean reversion, σ_t is the volatility, $\varepsilon_t \sim N(0,1)$ and T_{t-1}^m is the deterministic part of the temperature, which was modelled as

$$T_t^m = A + Bt + C\sin(\omega t + \varphi) \tag{3}$$

The term $C\sin(\omega t + \varphi)$ accounted for the seasonality in the data, while the term Bt accounted for the positive linear trend.

3. Computation of GDD

GDD is a measure of heat accumulation required for crops to develop, sprout new leaves, and attain reproductive stage and mature eventually (Yoshida, 1981; Basso & Naglekirk, 2014). It can be used in planning when to best plant and harvest the crops. Define the GDD per day, GDD_t , generated on that day, as

$$GDD_t = \max\{T_t - 10, 0\} \tag{4a}$$

$$\text{and the GDD for the whole 100-day period as } GDD = \sum_{i=1}^{100} GDD_i \tag{4b}$$

4. Computation volatility

Values for each day of every year for the past 59 years (1960 to 2018) were grouped making a total of 365 groups. Then the variance of each was computed, and a total of 365 variances were obtained.

5. Estimation of mean reverting speed

The formula given by Wang *et. al.* (2015) was used to obtain mean reverting speed:

$$\alpha = -\log \left(\frac{\sum_{i=1}^n \frac{T_{i-1} - T_{i-1}^m}{\sigma_{i-1}^2 (T_i - T_i^m)}}{\sum_{i=1}^n \frac{T_{i-1} - T_{i-1}^m}{\sigma_{i-1}^2 (T_{i-1} - T_{i-1}^m)}} \right) \tag{5}$$

6. Market Price of Risk and Risk-Free Rate

In this study, return on one-year term deposit of the Central Bank of the Philippines from 1987 to 2018 was used as risk-free rate, r . Then, using the return on Philippine Stock Exchange Index as the stock return, the following formula was used to compute the market price of risk λ

$$\lambda = \frac{\text{estimated } (\mu - r)}{\text{estimated } \sigma} = \frac{\frac{1}{n} \sum_{i=1}^n (k_i - r_i)}{\sqrt{\frac{1}{n-1} \sum_{i=1}^n ((k_i - r_i) - \frac{1}{n} \sum_{i=1}^n (k_i - r_i))^2}} \quad (6)$$

where k_i and r_i respectively represent the returns on stock and on term deposit in year t , $i = 1, 2, \dots, n$.

7. Pricing of European option

In pricing the European put option, the formula below was used.

$$p(t) = e^{-r(t_n-t)} \left[(K - \mu_n) \left(\Phi(\alpha_n) - \Phi\left(\frac{-\mu_n}{\sigma_n}\right) \right) + \frac{\sigma_n}{\sqrt{2\pi}} \left(e^{-\frac{\alpha_n^2}{2}} - e^{-\frac{1}{2}\left(\frac{\mu_n}{\sigma_n}\right)^2} \right) \right] \quad (7)$$

where, $\alpha_n = \frac{(K - \mu_n)}{\sigma_n}$, (8)

K is the strike price, and Φ denotes the cumulative distribution function for the standard Normal distribution. For the payout of the uncapped GDD European put option:

$$X = \text{amax}[K - G_n] \quad (9)$$

For a detailed derivation of the formula, one can refer to Alaton (2002) and Huang *et al* (2008).

SCOPE AND LIMITATIONS

The prices and payoffs presented in this study are applicable only for the province of Laguna and would need actual temperature reading from a weather station. With this, another limitation of this study is that some areas that are from the weather station can experience basis risk. Basis risk comes when the measurement device does not fully reflect the actual loss of the contract-holder. Basis risk can come from many sources such as lack of good density of weather station, a poor index design, and other reasons (Rao, 2010). In this form of contract, the basis risk is due to the lack of weather stations, which could have captured the real experience of the area.

Another limitation is that the design presented here is only for one unit of option. If the farmer wants more protection, then he could consider buying more than one unit of option.

Currently, there are also no existing market for options in the Laguna. However, the results of this study can open for new products that can be offered by agencies that already offer agricultural insurance products.

RESULTS AND DISCUSSION

Computation of parameters

Temperature data collected show that the highest and lowest temperature recorded in Laguna was 39°C, recorded on January 24, 1999 and 15°C, recorded on October 28, 2000, respectively. Employing equation 10, it was also found out that the maximum mean temperature and minimum mean temperature in Laguna from 1960 to 2018 was 32.2°C and 20.4°C, respectively. This range is good news for rice production since it is the range where rice crops grow the best (Yoshida, 1981). Data also shows that the coldest months are December, January, and February, while the hottest months are April, May, and June.

Figure 1 shows the daily actual mean temperature against the modelled deterministic part of

temperature (equation 10). According to the derived model, mean average temperature in Laguna shows an increasing trend. Particularly, it shows that there will be a 1.72°C increase in average temperature within 100 years from year 1960. This also suggests that there had been a 1.04°C increase in average temperature within the last 60 years. This result is close to the values (0.36°C in minimum temperature and 1.0°C in maximum temperature) PAGASA reported in 2011. The model also shows that the average temperature in Laguna from the year 1960 to the year 2018 was 26.88°C and that temperature in the area varied by 1.42°C from this average temperature. The regression model is shown in equation (10).

$$T_t^m = 26.88 + 4.74 \cdot 10^{-5} t - 1.42 \sin\left(\frac{2\pi}{365} t + 1.56\right) \tag{10}$$

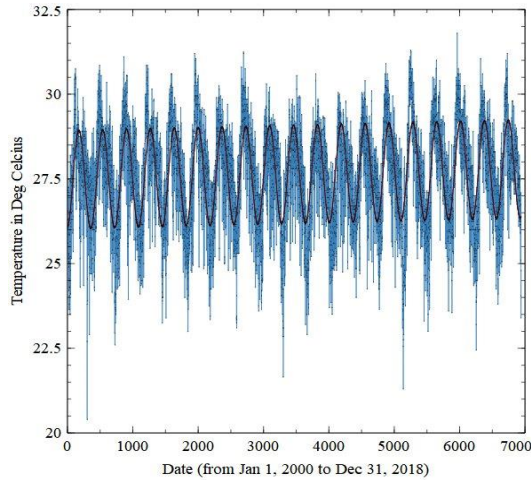


Figure 1. Daily Average Temperature in Laguna from 1960 to 2018 and Regression Model.

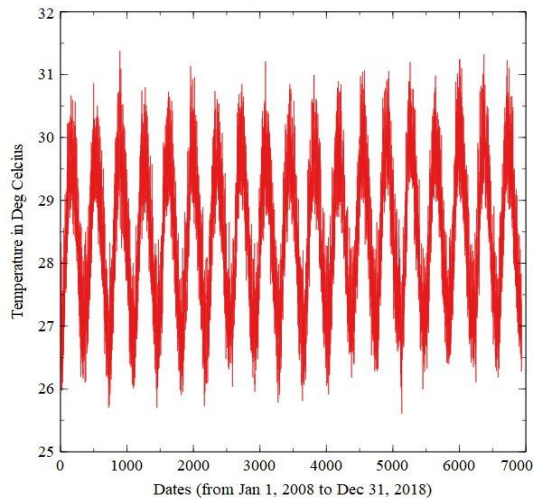


Figure 2. Stochastic Model of Daily Average Temperature in Laguna from 1960 to 2018.

Volatility was computed to be incorporated in the stochastic model and results are shown in Table 1. Results shows the month of January (1.58°C on the average) has the highest volatility. This implies that temperature on January tends to be more erratic (for example cold one day then hot the next day) than the other months. The mean reverting speed was found to be approximately $\alpha = 0.1907$. Figure 2 presents the results of the stochastic model.

To give a measure on how “good” the model is, the actual average temperature was compared to both the results of the deterministic model (equation 3) and the stochastic model (equation 2) and the computed root mean square error (RMSE) has a value of approximately 1.2196 and 1.6664, respectively. Measuring the performance of the temperature models, the mean percentage error (MPE) of actual mean temperature contrasted to the deterministic and stochastic models were roughly -0.0052 and -0.0054, respectively. This means that model for deterministic and stochastic models are over-forecasted by around 0.005%. No matter, these models can guide the options provider on pricing the insurance for a certain period.

Since there is a lack of an option market in the Laguna, the annual data acquired from Central Bank of the Philippines and Philippine Stock Exchange index from 1988 to 2018 were used to obtain the market price of risk and risk-free rate. The attained value of λ and r were approximately 0.0517 and 0.0720, respectively.

Pricing of the option

From the results above, the price of the option was obtained. Using 12°C as the base temperature, different growing degree days as the strike value, and 100 contract days the price of the GDD European put option are obtained. Table 2 shows that price per month upon choosing a specific GDD strike value. The strike values in Table 2 were chosen since according to Yoshida (1981), the best temperature to grow rice is between 22°C to 32°C. These values generate a GDD of 1000 to 2000, thus the choice of strike values from 1500 to 2000. There are blanks in Table 2 since during his period for a certain strike value, the price of the option is zero. This means that risk of loss during that time is very small and there is no need to buy the option to protect against such loss.

To buy the option, the buyer would need to choose first what strike value he would be buying the option at. Strike values are GDD values. Buying the option at 1500 would mean that the buyer expects that the GDD during the contract time (the 100-day period) would be below 1500, causing the crops to not grow well. If the GDD during the 100-day period hit above the strike value, then the buyer would receive nothing at the end of the contract. However, if the actual GDD is lower, then the buyer would receive the difference of actual GDD and the strike value at the end of the contract period, if he chooses to exercise the option. Naturally, the buyer would want to buy a contract with a higher GDD, however, this would mean that he would need to pay more for it. Thus, it would be best to choose the strike value wisely.

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Table 1. Volatility of temperature per day.

Days	Months											
	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
1	1.195	1.783	1.259	1.06	0.804	0.813	1.043	1.128	1.276	0.734	1.597	1.115
2	0.927	1.458	1.323	0.99	1.106	1.14	0.811	1.087	0.972	0.825	1.416	1.002
3	0.809	1.91	1.393	1.817	0.662	1.078	0.732	1.144	1.278	0.754	1.325	1.326
4	1.12	1.722	1.614	1.505	0.587	1.182	0.692	1.262	1.045	1.031	1.641	1.403
5	1.033	1.66	2.083	1.301	0.704	1.029	1.623	1.095	1.372	1.191	0.981	1.629
6	1.127	2.3	1.701	0.686	1.083	1.455	0.978	1.418	1.01	0.996	1.256	1.612
7	1.331	1.891	1.546	0.821	1.175	0.955	0.952	1.543	0.873	0.888	1.013	1.309
8	1.506	1.387	1.724	0.746	1.047	1.123	0.763	1.232	1.135	1.029	0.751	1.429
9	2.083	1.569	1.481	0.996	1.358	1.07	0.982	1.363	1.165	0.889	0.912	1.083
10	1.711	1.437	1.556	0.793	1.192	1.177	0.96	1.586	1.1	0.866	0.939	1.556
11	1.932	1.457	1.406	1.081	0.902	1.214	0.87	1.175	0.788	0.891	1.367	1.51
12	1.249	0.964	1.385	0.846	1.208	1.196	1.031	0.848	0.865	1.334	1.125	1.315
13	2.326	1.474	1.347	1.173	1.401	1.096	1.473	1.027	0.935	1.584	1.28	1.313
14	2.159	1.149	1.273	1.249	1.274	0.759	1.43	1.568	0.894	0.881	1.098	1.54
15	1.746	0.922	1.29	0.81	0.971	1.263	1.217	1.348	1.008	1.009	1.251	1.44
16	1.492	1.25	1.432	0.989	1.249	0.954	1.057	1.158	0.907	1.038	1.181	1.474
17	1.672	1.558	1.287	0.874	1.476	0.876	1.307	1.444	0.807	0.975	1.31	1.355
18	1.569	1.558	1.262	0.771	1.657	0.979	1.251	1.33	1.088	0.988	0.992	1.636
19	1.585	1.402	1.129	0.779	1.499	1.173	1.106	1.491	1.002	1.129	1.399	1.222
20	1.709	1.585	1.2	1.05	1.227	0.997	1.754	1.272	0.762	1.069	1.494	1.245
21	1.395	1.384	1.174	1.282	1.837	0.842	1.332	0.941	0.904	1.482	1.1342	1.859
22	1.661	1.3364	1.0299	0.7413	1.5684	0.9293	1.3015	0.9446	0.9896	1.331	1.409	1.346
23	1.88	1.376	1.046	1.239	1.334	1.376	1.021	1.026	0.869	0.878	0.791	1.667
24	1.869	1.084	1.127	0.91	1.183	1.622	1.167	1.256	0.962	1.119	0.983	1.953
25	1.13	1.366	1.172	0.787	1.495	1.578	1.211	0.905	0.884	1.339	1.08	1.463
26	1.55	1.533	1.233	0.703	1.755	1.471	1.053	1.331	1.161	0.95	1.105	1.042
27	1.426	1.281	1.246	0.714	1.815	1.494	1.17	1.347	0.901	1.24	1.387	1.62
28	1.975	1.655	1.183	0.682	1.365	1.038	0.84	1.151	0.884	1.687	1.677	1.932
29	1.901		1.1	0.853	1.103	1.174	0.862	1.355	1.149	1.247	1.059	1.571
30	1.922		0.82	1.182	1.202	1.171	0.894	1.158	0.803	0.839	1.089	2.241
31	2.003		0.982		1.224		1.125	0.867		1.117		1.213

For a more concrete example, if the farmer wanted to hedge against low temperature and acquired the one unit of GDD European put option on September 1, usual transplanting month in some areas of Laguna, and chose a strike value of 1800 GDD. He would need to pay 0.13 PhP. Then, at the end of the 100-day period, the actual GDD was found to be 1478.92, which was below the strike value of 1800 GDD. Under the assumption that the best GDD to grow crops at was 1800, then farmer could have experienced yield loss and therefore, could have experienced financial loss as well. If the farmer exercised the option, then he would have received a payout of 321.08 PhP.

Table 2. Price of the GDD European put option (in PhP) per month for October 2019 to September 2020.

Month	Strike Values					
	1500	1600	1700	1800	1900	2000
January		0.02	0.10	0.17	0.25	0.32
February			0.05	0.12	0.20	0.27
March				0.08	0.15	0.23
April				0.04	0.11	0.19
May				0.01	0.09	0.16
June				0.01	0.09	0.16
July				0.04	0.11	0.19
August			0.01	0.08	0.16	0.23
September			0.06	0.13	0.20	0.28
October		0.04	0.12	0.19	0.27	0.34
November	0.50	0.57	0.64	0.71	0.78	0.85
December	0.50	0.56	0.63	0.71	0.78	0.85

CONCLUSION AND RECOMMENDATION

This study presents a way to protect the farmers from financial consequences due to natural disasters. It was shown in the previous section that in the Philippines, specifically in the province of Laguna, temperature only varies from around 20 °C to 30 °C. This low variation of temperature resulted to lower volatility and, hence, lower price of the option. This is good news since this new product would be more affordable for the farmers, who would be needing financial protection more in the future since due to climate change temperature is expected to be volatile and intense. Strike values were also chosen based on the critical high and low temperature were crops grown. It is up to the farmer to choose among the provided strike values. The price of the option was also found to vary from 0 to 1 PhP, which is relatively cheaper than other agricultural financial instruments. The results of this study can help insurance providers and the government to design products that can help the farmers.

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STATEMENT OF AUTHORSHIP

The first and second authors gathered the data, finalized the model and design of put option, and prepared the initial manuscript. The third author constructed the problem and wrote the final manuscript. The fourth and fifth authors did the initial model fitting and design.

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